Pertanika J. Sci. & Technol. **15(1)**: 25 - 34 (2007)
ISSN: 0128-7680
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Array Beampattern Using Goal

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Received: 2 August 2004

ABSTRAK

Corak pancaran di kedudukan jauh daripada punca bunyi boleh dibentuk dengan menukarkan amplitud dan fasa setiap kesusunan antena. Walau bagaimanapun, apabila satu atau lebih elemen gagal, corak pancaran akan terganggu. Algoritma 'Goal Attain' boleh digunakan untuk memperbaiki corak tersebut dengan mengoptimasikannya supaya sama dengan corak pancaran yang asal. Ini boleh dicapai dengan mengubah dan mengawal fasa serta amplitud setiap elemen beberapa kali. Kualiti corak yang dibaiki boleh dibandingkan dengan corak yang asal dengan satu indeks kecapaian yang tertentu. Kebaikan Goal Attain telah dikaji dalam sistem bunyi 'Uniform Planar' dan 'Linear Array'.

ABSTRACT

The desire beampattern of a sound source at the far field can be synthesised by controlling the relative amplitudes and phases of each array element. However, when one or more elements in the array fail, the beampattern at the far field is severely distorted. The Goal Attain optimisation algorithm is used to adjust the relative gains of the remaining array elements so as to generate an optimised pattern closely resembles the original beampattern. The quality of each optimised pattern is evaluated and compared based on a defined performance index. The effectiveness of the Goal method is tested in both the Uniform Planar and Linear Array sonar systems.

Keywords: Beampattern, beamwidth, linear array, planar array, goal attain optimisation

INTRODUCTION

The desire shape of the far field radiation pattern of a linear uniform array antenna system can be synthesised by controlling the relative amplitudes and phase of the array element weights.

The array antennas typically consist of individual elements that are essentially each little antenna on their own. Numbers of these elements are arrayed in patterns depending on the desired performance characteristics needed by the application such as operating frequencies, antenna gain, sensitivity, and power requirements. When one or more array elements or its associated circuitries of the antenna system become defective, the result, in severe distortion of the original far-field pattern. If all array elements operate properly, the well-known analytic techniques can be used to find the optimum amplitude and phase of each element to yield the desired pattern. However, if any elements fail, no analytic means exists to find the excitation coefficients of the array that compensates for the degradation of the pattern.

Recently, there has been an increased interest in developing methods to improve the patterns of arrays in the presence of failed elements by re-optimising the weights applied to the remaining array elements. Peters (1991) proposed a method to reconfigure the amplitude and phase distortion of the remaining elements by minimizing the average sidelobe level via Conjugate Gradient Method. Er (1992) matched the array response

over the main lobe width while minimizing the mean-square value of the array response over the sidelobe regions. This formulation results in a constrained optimization problem involving a quadratic constraint and a set of linear constraints on the element excitation. Bu *et al.* (1993) used the biquadratic programming method in reconfiguring the array by changing the phase of each remaining active element. Maxillous (1996) used the method of replacing the signals from failed elements in a digital beamforming receiving-array. Levitas *et al.* (1999) introduced a practical failure compensation technique for active phasing arrays. It is independent of the external signal environment and is capable of achieving substantial performance improvement across broad selectable angular sectors at the expense of some additional performance degradation in other less important sectors. Yeo *et al.* (1999) proposed the genetic algorithms for array failure correction in digital beamforming of arbitrary arrays. Deng *et al.* (2001) applied the optimisation algorithm, differential evolution to the radiation pattern synthesis of the array. Miaris *et al.* (2002) uses the orthogonal method to find the excitation of each non-defective element factor and considers only the array factor established with orthogonal basis.

This paper describes Goal Attain technique to restore the original far field radiation pattern by adjusting the gains of the remaining working elements. The Goal Attain technique is more relevant when dealing with faulty array elements since the Goal objective is chosen to restore maximally possible the correct beampattern to avoid degradation in performance of the array system. To use Goal Attain, two beam patterns are required; the original and the distorted pattern caused by faulty elements. The original beampattern is determined by numerically integrating the gains and phases of all the working elements in the sound source array. To synthesise the fail beam pattern, the phases and gains of the faulty array elements are null during the integration process. The effect of the faulty array elements on the beampattern is the overall reduction in the array gain factor. In addition, lobe symmetry is also affected in absence of non-coherent inter-element spacing between elements *(Fig. 1)*. The original beampattern is set as the target in Goal Attain. The Goal Attain then attempt to minimize the error between the faulty and the original pattern by adjusting the gain (weights) of the remaining working elements in the array.

The availability of powerful Digital Sequel Processors (DSPs) and their efficient computation algorithms can be exploited to implement optimisation techniques in real time sonar array systems. Although the Goal Attainment optimisation technique is applied to linear and planar arrays in this paper, the procedure can be generalized for other array configurations.

FAR FIELD BEAMPATTERN GENERATION OF SOUND SOURCE ARRAY – BEAMFORMING

The linear and planar arrays considered in this work (Wykes *et al.*, 1992; Wykes and Nagi, 1992) operate at Ultrasonic frequency *of* f_c = *100KHz* and the speed of sound in air at 20º C is c = *344 m/sec*; the inter-element distance '*d*' avoiding grating is

$$\lambda = \frac{c}{f_c} \Rightarrow \frac{344}{100{,}000} = 3.44mm$$

$$d \leq \frac{\lambda}{2} = 1.72mm$$

The linear array consists of *M=16* where as planar consists of *MxN (8x8)* elements. The resultant sound field at a particular far field point is determined by the vector addition of the sound field at that point radiated by the individual sound sources. Conventional beamforming calculates the time delay of the signals arriving between array elements by shifting and summing the incoming data using numerical integration.

Linear Array

The array factor of the beampattern is normalised with equal amplitudes *w(m)* of one. The beamforming output from linear array is given as

$$G(\theta) = \sum_{m=1}^{M} w(m) e^{i(2m-M-1)*arg} \tag{1}$$

where

$$arg = \pi d \left[sin\left(\theta - \frac{\pi}{2} - \gamma\right)\right]$$

θ is scanning angle, q = *-90 :1:90*
γ is steering angle
w(m)=[1,1,.....,M], Uniform weight on each element
d, inter-element distance between array elements
M=16 array elements *and m* is the index *on M*

The quantity $|G(\theta)|^2$ is known as the beampattern, which is simply the power of the beamformer as a function of θ.

Planar Array

Planar array beampattern G_p in two-dimensional is superposition of two beampatterns S_x in x-axis and S_y in y-axis

$$G_p = S_x(\theta,\varphi) * S_y(\theta,\varphi) \tag{2}$$

where

$$S_x(\theta,\varphi) = \sum_{m=1}^{M} w_x(m) e^{i(2m-M-1)*arg_x}$$

$$arg_x = \pi d_x \left[cos\left(\theta - \frac{\pi}{2} - \gamma_x\right) sin\left(\phi - \frac{\pi}{2} - \gamma_y\right)\right]$$

θ and φ =-90:1:90, are the scanning in x and y dimension

and

$$S_y(\theta,\varphi)=\sum_{n=1}^{N} w_y(n)e^{i(2n-N-1)*arg_y}$$

$$arg_y=\pi d_y\left[sin\left(\theta-\frac{\pi}{2}-\gamma_x\right)sin\left(\phi-\frac{\pi}{2}-\gamma_y\right)\right]$$

γ, θ, φ and $w_x(m) = w_y(n) = w(m)$ are same as before.

FAILED BEAMPATTERN

To simulate the beampattern due to failed elements, equations (1) and (2) are used. A new weight vector $w_f(q)$ is used in equations (1) and (2) instead of $w(m)$ which has zero at the positions of the failed elements.

$w_f(q)$ = [1,0,1,0,0,1,1.....Q= M-P] Array element # 2,4,,5.....failed (3)

where Q is the number of the remaining working elements and q is their index. P represent the numbers of the failed elements and p is the index of the failed elements. The beampattern of the remaining elements excluding the contribution from the failed elements is evaluated as

$$G_f(\theta)=\sum_{q=1}^{Q} w_f(q)e^{i(2(q+p)-Q-1)*arg} \quad (4)$$

In equation (4), index q+p cater for increased inter-element distance in multiple of d, as in *Fig. 1*. The purpose of $w_f(q)$ and $(q+p)*d$ is to avoid contributions from the failed elements

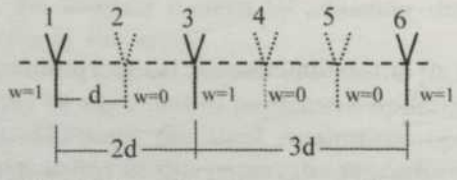


Fig. 1: Non-uniform inter-element distance

$$(q+p)*d = (1, 2, 3..)* d \quad (5)$$

In the planar array system, the randomly selected elements are also failed similarly in *Fig. 2*. The weights w_x and w_y are changed to w_{fx} and w_{fy} as in equation (3).

$$G_{pf}(\theta,\varphi)=\left[\sum_{u=1}^{U} w_{fx}(u)e^{i(2(u+p)-U-1)*arg_x}\right]\left[\sum_{v=1}^{V} w_{fy}(v)e^{i(2(v+p)-V-1)*arg_y}\right] \quad (6)$$

where u, v and U, V are the indexes and numbers of the failed x-y planar array respectively, similar to q and Q in equation (5) for failed linear array.

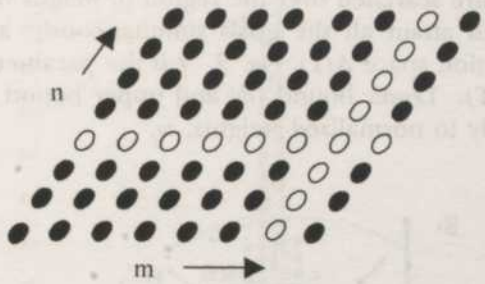


Fig. 2: Failed elements of planar array are shown white

OPTIMISATION OBJECTIVE FUNCTION AND PATTERN

The optimisation process evaluates the optimised weights $w_o = [w_{o1}\, w_{o2} \ldots .. \, w_M]$ compensating the failed elements weights w_f. The failed element beampattern $G_f(w_f)$ is chosen as the optimisation objective function so that during the optimisation process, the failed array elements do not contribute to the optimised beampattern. Goal Attain then iteratively evaluates the weights of all the working elements in the array. Each weight w_{op} consists of the relative amplitude of each working element The weights are so adjusted such that $G_f(w_f)$ is optimised to $G_f(w_o)$ and approaches closest to the goal $G(q)$. The optimised objective function is described here in the with linear array but it can be extended similarly to planar array. The optimised beam patterns can then be described as

$$G_f\left(w_f = w_0\right) = \sum_{m=1}^{M} w_o(m) e^{i\left(2(m+p)-M-1\right)^* arg} \tag{7}$$

and for planar array

$$G_f(\theta,\varphi)_o = Sx(\vartheta,\varphi)o \; * \; Sy(\theta,\varphi)_o \tag{8}$$

OPTIMISATION – GOAL ATTAINMENT

Multi-objective optimisation method of Gembicki (1974) concerns with the minimization of a set of objectives simultaneously. According to Hiller and Lieberman (2001) "The basic idea is to establish a numerical goal for each of the objectives, formulate an objective function for each objective and seek a solution from their respective goals". Mathematically, for array gain optimisation, the problem can be stated

$$\underset{\gamma,\, w \in \Omega}{\text{Minimize}} \quad \Theta * \gamma = G_f(w_f) - G^*$$

$$\text{or} \quad G_f(w_f) - \Theta * \gamma \leq G^* \tag{9}$$
$$0\ (lb) \leq w_f\ 1\ (ub)$$

$G^* = [g_1^*, g_2^*, \ldots g_m^*]$ are goals to be achieved by set of objective $G_f = [g_{f1}(w_f), g_{f2}(w_f), \ldots g_{fm}(w_f)]$. Where g^* is described as isotropic field of radiation of individual array element and g_f is describe as radiation pattern as a function of w_f. Each objective function g_f and its corresponding goal g^* are searched over the region of weight vector $[w_{f1}\, w_{f2} \ldots .. w_{fM}]$. Since it is not possible to attain all the goals simultaneously, a feasible region Ω is explicitly defined by function space $\Lambda(\Upsilon)$. *Fig. 3*, γ *is* the parameter which changes the size of function space $\Lambda(\Upsilon)$. Lower bound *(lb)* and upper bound *(ub)* are constraint in (9) to 0 and 1 respectively to normalized weights, w_f

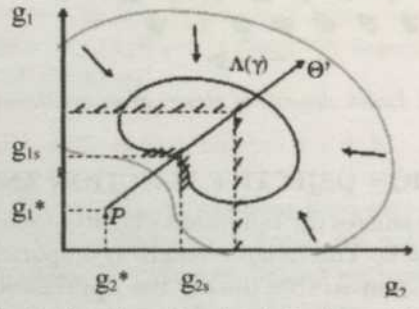


Fig. 3: Two dimensional optimisation problem

The sign of Θ determines the direction of search of the feasible function space from goal point P g_1^*, g_2^*. The +ve of Θ in equ.(9) makes the $G_f(w_f)$ less than G^*, while -ve makes $G_f(w_f)$ greater than G^* and function converges closer to the solution The starting weights w_f *(M)* are carefully chosen so as not to produce local minima The starting optimisation function $|G_f(w_f)|$ then would be less than $|G^*|$, so

$$w_f = -(1 \text{ to } 2)$$

The magnitude of 1 to 2 is selected for the slackness in the solution and helps to increase speed of convergence. After optimisation the optimised weights are then

$$w_{opt} = (w_f)^*$$

PERFORMANCE MEASURE

After the optimised pattern has been determined, it is compared with the failed beampattern to evaluate if any improvement in performance has occurred. Gain performance indicator η (SNR) is set to the ratio of the beamwidth defined as the gain of the main beam above -3db, to the side lobes as

$$SNR \Rightarrow \eta_{(f,opt)} = 20 log_{10}\left[\frac{\Sigma G_{BW_{3db}}}{\Sigma\left[G_f(w_o) - G_{BW_{3db}}\right]}\right]$$

Fig. 4 depicts the above expression. The rationale for η is that the main beam is pointing towards the desired direction and the side lobes give rise to undesirable noise entering in the receiver from other undesirable direction.

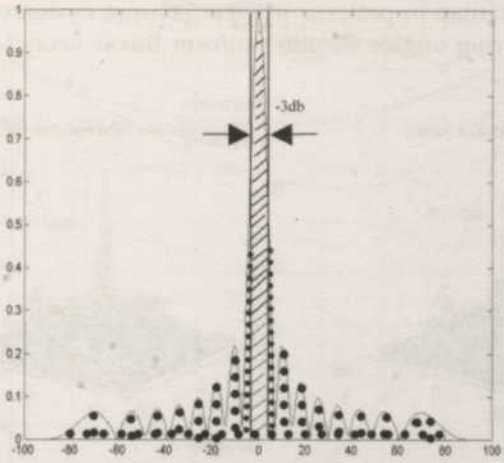


Fig. 4: 3db beamwidth used in equ. (9)

RESULTS

Sixteen element uniform linear array optimisation results were evaluated at different steering angles with different positions of failed elements. Matlab's optimization toolbox (1996) 'fgoalattain' command was used for implementation of algorithm in section 5. *Figs. 5-6* illustrate the beampattern results. The reference pattern $|G(q)|$ is obtained equations (1) and (2), failed patterns $|G_f(q)|$ from equations (4) and (6), optimised beampattern $|G_{opt}(q)|$ from equations (7) and (8). The selected failed elements in equation (2) are set as '0.000' in the weight column at the left of *Fig. 5.*

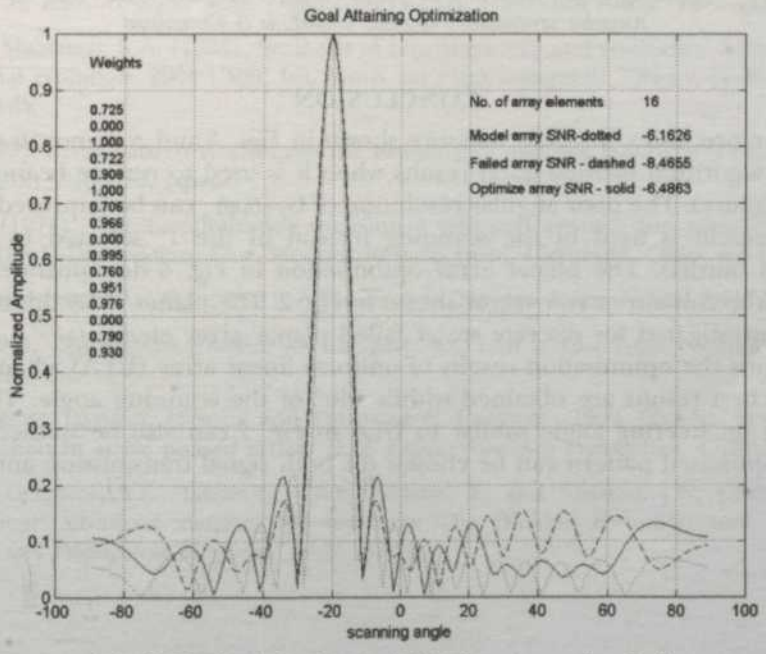


Fig. 5: Goal optimisation for failed elements numbers 2, 9 and 14. Element separation d=1.72mm

The η of the three-radiation patterns in *Figs. 5-6* were evaluated using equation (10). The η at different steering angles for the uniform linear array is shown in *Fig. 7.*

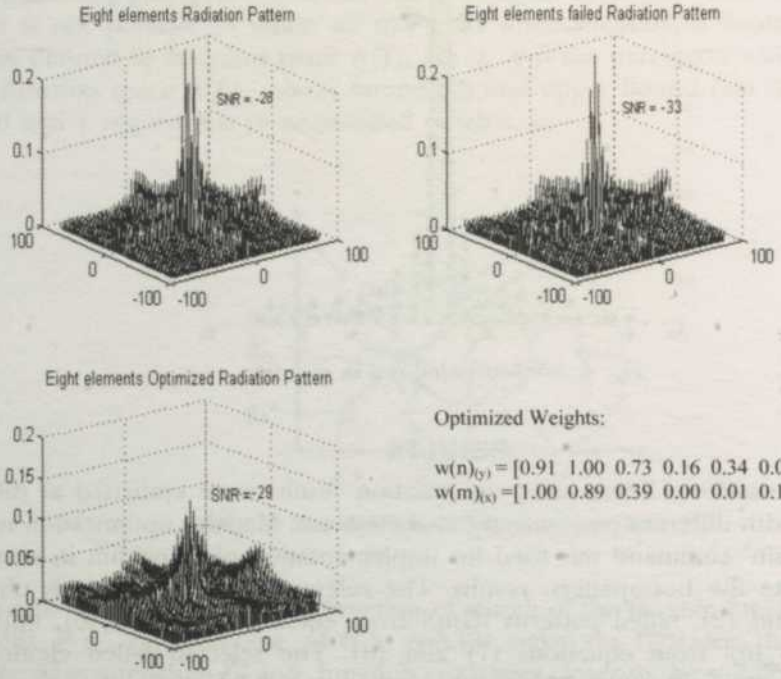


Fig. 6: Failed element vector three or x-axis and vector seven on x-axis, see fig. 2. Scanning angles theta and psi range –90 to 90 degrees. SNR = η, Angular resolution 5 degrees, amplitude is normalized

CONCLUSION

The performance index η (SNR) measure shown in *Figs. 5* and *6* demonstrates that the Goal Attain algorithm provide good results when it is used to restore beampattern due to element failures. The poor angular resolution of $G_f(\theta,\varphi)_o$ can be improved if a smaller degree increment is used in the scanning instead of the 1° selected to reduce the computation burden. The planar array optimisation in *Fig. 6* demonstrates the failed elements in the column or row vector shown in *Fig. 2.* The planar array optimisation can be further investigated for discrete scalar failed planar array elements.

Fig. 7 shows the optimisation results of uniform linear array (ULA) when steered off centre. The best results are obtained within ±40° of the scanning angle. The effect of optimisation on steering angle similar to ULA in *Fig. 7* can also be studied for planar array. The optimised pattern can be chosen for both signal transmission and reception.

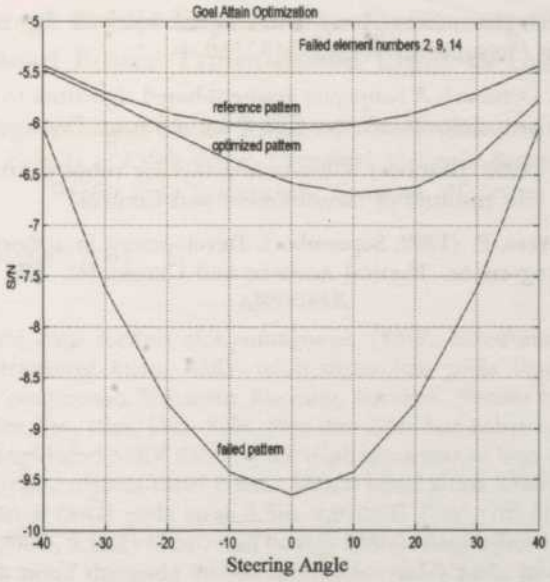


Fig. 7: h (SNR) vs. scanning angle plot of uniform linear failed elements array in Fig. 5

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